

# YEAR 9 — REASONING WITH ALGEBRA... Straight Line Graphs

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use  $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

## Keywords

**Gradient:** the steepness of a line

**Intercept:** where two lines cross. The y-intercept: where the line meets the y-axis

**Parallel:** two lines that never meet with the same gradient

**Co-ordinate:** a set of values that show an exact position on a graph

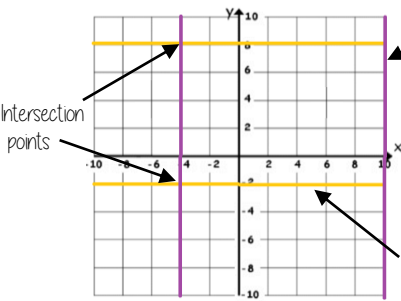
**Linear:** linear graphs (straight line) — linear common difference by addition/ subtraction

**Asymptote:** a straight line that a graph will never meet

**Reciprocal:** a pair of numbers that multiply together to give 1

**Perpendicular:** two lines that meet at a right angle

## Lines parallel to the axes



All the points on this line have a x coordinate of 10

Lines parallel to the y axis take the form  $x = a$  and are vertical

Lines parallel to the x axis take the form  $y = a$  and are horizontal

All the points on this line have a y coordinate of -2  
eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

'a' can be ANY positive or negative value including 0

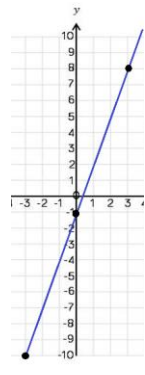
## Plotting $y = mx + c$ graphs

$y = 3x - 1$  → 3 x the x coordinate then - 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

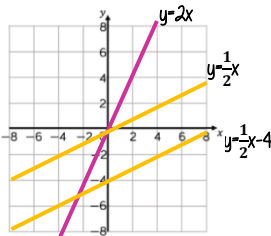
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

## Compare Gradients

$y = mx + c$

The coefficient of x (the number in front of x) tells us the gradient of the line



The greater the gradient — the steeper the line

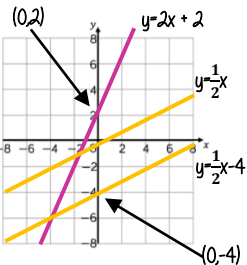
Positive gradients

Parallel lines have the same gradient

Negative gradients

## Compare Intercepts

$y = mx + c$  ← The value of c is the point at which the line crosses the y-axis Y intercept



The coordinate of a y intercept will always be (0,c)

Lines with the same y-intercept cross in the same place

$y = mx + c$

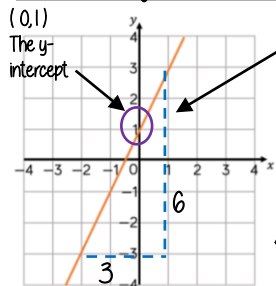
The coefficient of x (the number in front of x) tells us the gradient of the line

$y = mx + c$   
y and x are coordinates

The value of c is the point at which the line crosses the y-axis Y intercept

The equation of a line can be rearranged. Eg  
 $y = c + mx$   
 $c = y - mx$   
Identify which coefficient you are identifying or comparing

## Find the equation from a graph



The Gradient  $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

## Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

The y-intercept shows the minimum charge.  
The gradient represents the price per mile

## Direct Proportion graphs

To represent direct proportion the graph must start at the origin

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

When you have 0 pens this has 0 cost.  
The gradient shows the price per pen

# YEAR 9 — REASONING WITH ALGEBRA...

## Forming and Solving Equations

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### What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

### Keywords

**Inequality:** an inequality compares two values showing if one is greater than, less than or equal to another

**Variable:** a quantity that may change within the context of the problem

**Rearrange:** Change the order

**Inverse operation:** the operation that reverses the action

**Substitute:** replace a variable with a numerical value

**Solve:** find a numerical value that satisfies an equation

### Solve equations with brackets



$$3(2x + 4) = 30$$

$$6x + 12 = 30$$

$$6x = 18$$

$$x = 3$$

$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$6x = 18$$

$$x = 3$$

### Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

$$3x + 2 > 11$$

Solve

$$x > 3$$

### Inequalities with negatives

**Method 1** Make x positive first

$$2 - 3x > 17$$

$$+ 3x \quad + 3x$$

$$2 > 17 + 3x$$

$$-17 \quad -17$$

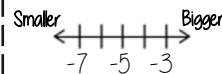
$$-15 > 3x$$

$$\div 3 \quad \div 3$$

$$-5 > x$$

x is true for any value smaller than -5

✓ CHECK IT!  
 $2 - 3(-6) = 20$   
TRUE/ CORRECT



### Equations with unknown on both sides

$$4x + 5 = 3x + 24$$

$$-3x \quad -3x$$

$$x + 5 = 24$$

$$-5 \quad -5$$

$$x = 19$$

$$x \quad x \quad x \quad x \quad 5$$

$$x \quad x \quad x \quad 24$$

### Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$$5(x + 4) < 3(x + 2)$$

$$5x + 20 < 3x + 6$$

$$2x + 20 < 6$$

$$2x < -14$$

$$x < -7$$

$$5(-8 + 4) < 3(-8 + 2)$$

$$5(-4) < 3(-6)$$

$$-20 < -18$$

✓ -20 IS smaller than -18

Check it!

**Method 2** Keep the negative x

$$2 - 3x > 17$$

$$-2 \quad -2$$

$$-3x > 15$$

$$\div -3 \quad \div -3$$

$$x > -5$$

x is true for any value bigger than -5

This cannot be true...

$$x < -5$$

When you multiply or divide x by a negative you need to reverse the inequality

### Formulae and Equations

Substitute in values

Formulae — all expressed in symbols

Equations — include numbers and can be solved

### Rearranging Formulae (one step)

$$x = y + z$$

$$x = y + z$$

Rearrange to make y the subject.

$$y = x - z$$

$$y \rightarrow +z \rightarrow x$$

$$y \leftarrow -z \leftarrow x$$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging...

Make XXX the subject

Change the subject

Rearrange

### Rearranging Formulae (two step)

In an equation (find x)

$$4x - 3 = 9$$

$$+3 \quad +3$$

$$4x = 12$$

$$\div 4 \quad \div 4$$

$$x = 3$$

In a formula (make x the subject)

$$xy - s = a$$

$$+s \quad +s$$

$$xy = a + s$$

$$\div y \quad \div y$$

$$x = \frac{a + s}{y}$$

The steps are the same for solving and rearranging

Rearranging is often needed when using  $y = mx + c$

e.g Find the gradient of the line  $2y - 4x = 9$

Make y the subject first  $y = \frac{4x + 9}{2}$

$$\text{Gradient} = \frac{4}{2} = 2$$

# YEAR 9 — REASONING WITH ALGEBRA...

## Testing conjectures

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### What do I need to be able to do?

By the end of this unit you should be able to:

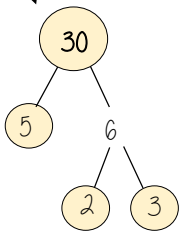
- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

### Keywords

- Multiples:** found by multiplying any number by positive integers
- Factor:** integers that multiply together to get another number.
- Prime:** an integer with only 2 factors.
- HCF:** highest common factor (biggest factor two or more numbers share)
- LCM:** lowest common multiple (the first time the times table of two or more numbers match)
- Verify:** the process of making sure a solution is correct
- Proof:** logical mathematical arguments used to show the truth of a statement
- Binomial:** a polynomial with two terms
- Quadratic:** a polynomial with four terms (often simplified to three terms)

### Factors, Multiples and Primes

Multiplication part-whole models



All three prime factor trees represent the same decomposition

**HCF – Highest common factor**

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors are factors two or more numbers share

**LCM – Lowest common multiple**

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

Common multiples are multiples two or more numbers share



### True or False?

Conjecture

A pattern that is noticed for many cases

1, 2, 4, ...  
The numbers in the sequence are doubling each time.

Counterexamples



This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

### Always, Sometimes, Never true.

**Always** Every value always supports the statement

**Sometimes** Examples show the statement being true and counter examples to show when it is false.

**Never** No example supports the statement

Examples to try

- 0 and 1
- Fractions
- Negative numbers

### Show that

Numerical verification

Show the stages to a solution with numerical values

Algebraic verification

Show algebraic properties of the solution  
You may want to use pictorial images to support this

Proof

Simple proofs using algebra

Compare the left hand side of an equation with the right hand side – are they the same or different?

### Conjectures



Even  
(2n)  
Multiple of 2



Odd  
(2n + 1)  
One more than any even

Use numerical verification first  
Use pictorial verification – the representations of numbers of odd and even

### Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers

Positions of numbers in relation to n form expressions

Eg one space to the right of n  
 $n + 1$

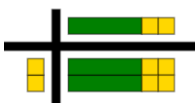
Eg One row below n  
 $n + 10$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

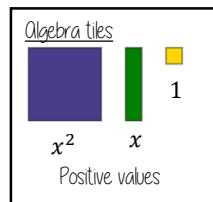
The size of the grid for generalisation changes the relationship statements

### Expanding binomials

$$2(x + 2) \equiv 2x + 4$$

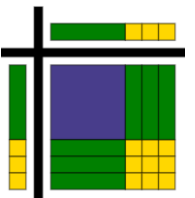


Algebra tiles can represent a binomial expansion  
Has two terms



The order of the binomial has no impact on the outcome.  
eg  $(x + 3)(3 + x)$

$$(x + 3)(x + 3) \equiv x^2 + 6x + 9$$



This is a quadratic  
It has four terms which simplified to three terms

# YEAR 9 — CONSTRUCTING IN 2D/3D... 3D Shapes

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## What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

## Keywords

**2D:** two dimensions to the shape e.g length and width

**3D:** three dimensions to the shape e.g length, width and height

**Vertex:** a point where two or more line segments meet

**Edge:** a line on the boundary joining two vertex

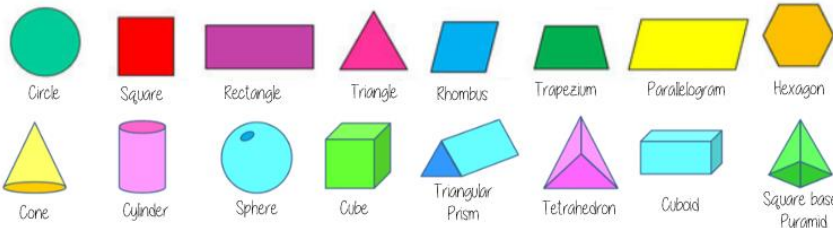
**Face:** a flat surface on a solid object

**Cross-section:** a view inside a solid shape made by cutting through it

**Plan:** a drawing of something when drawn from above (sometimes birds eye view)

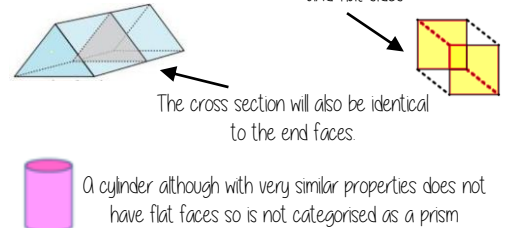
**Perspective:** a way to give illustration of a 3D shape when drawn on a flat surface.

## Name 2D & 3D shapes

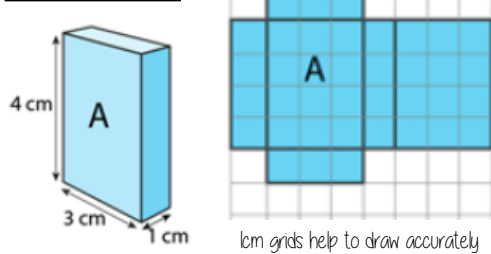


## Recognise prisms

A solid object with two identical ends and flat sides

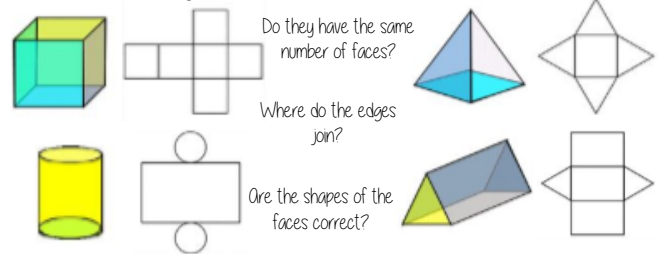


## Nets of cuboids

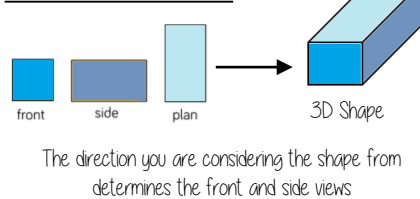


Visualise the folding of the net. Will it make the cuboid with all sides touching

## Sketch and recognise nets

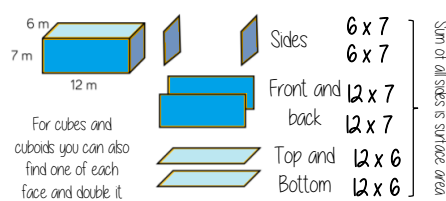


## Plans and elevations



## Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area



For other shapes - not all the sides are the same, so calculate the individually

## Volumes

Volume is the 3D space it takes up - also known as capacity if using liquids to fill the space

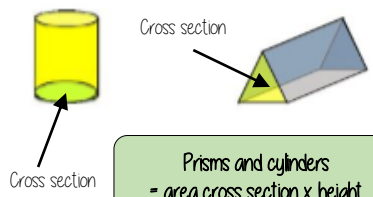


### Counting cubes

Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

**Cubes/ Cuboids = base x width x height**

Remember multiplication is commutative

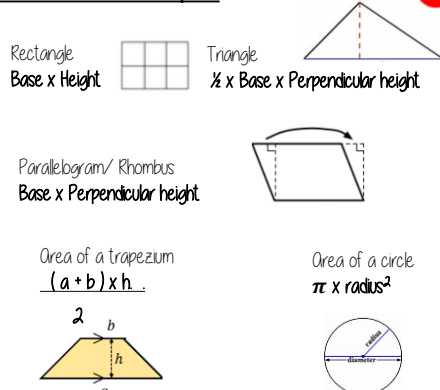


Height can also be described as depth

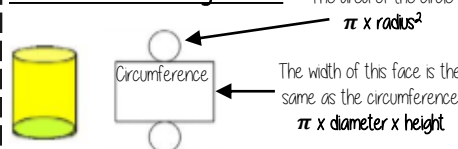
Areas - square units  
Volumes - cube units

Areas and volumes can be left in terms of  $\pi$

## Area of 2D shapes



## Surface area - cylinders



**$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$**

# YEAR 9 — CONSTRUCTING IN 2D/3D...

## Constructions & congruency

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### What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

### Keywords

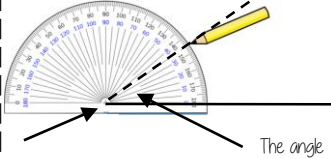
- Protractor:** piece of equipment used to measure and draw angles
- Locus:** set of points with a common property
- Equidistant:** the same distance
- Discorectangle:** (a stadium) — a rectangle with semi circles at either end
- Perpendicular:** lines that meet at  $90^\circ$
- Arc:** part of a curve
- Bisector:** a line that divides something into two equal parts
- Congruent:** the same shape and size

### Draw and measure angles

R

Draw a  $35^\circ$  angle

Make a mark at  $35^\circ$  with a pencil and join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

### Scale drawings

R

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

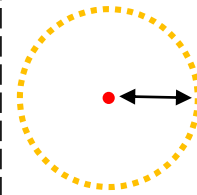
The car image is 10cm



Image: Real life  
1cm : 30cm  
 $\times 10$   $\leftarrow$   $\rightarrow$   $\times 10$   
10cm : 300cm

### Locus of a distance from a point

All points are equidistant (the same distance) from the fixed point in the middle



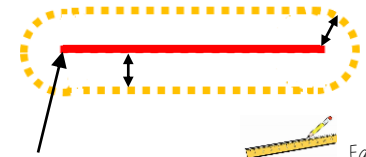
If the point is in the corner it can only make a quarter circle



Equipment needed  
The radius is the distance from the fixed point

### Locus of a distance from a straight line

All points are equidistant (the same distance) from line



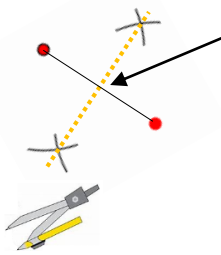
The ends of the line are fixed points



Equipment needed  
The line is straight so a ruler is used for the straight lines parallel to your original line

### Locus equidistant from two points

Also a perpendicular bisector  
Because if the points are joined this new line intersects it at a  $90^\circ$



Join the intersections with a ruler.  
All points on this line are equidistant from both points

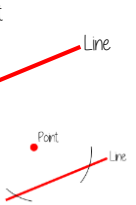


Keep the compass the same size and draw two arcs from each point

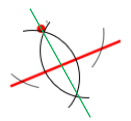
### Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



Keep the compass the same distance and now use your new points to make new intersecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

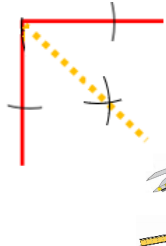
### Locus of a distance from two lines

Also an angle bisector  
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

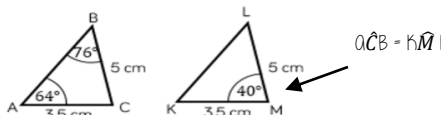


### Congruent figures

Congruent figures are identical in size and shape — they can be reflections or rotations of each other



Congruent shapes are identical — all corresponding sides and angles are the same size



Because all the angles are the same and  $AC=KM$   $BC=LM$  triangles ABC and KLM are congruent

### Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

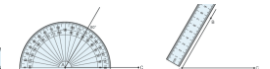
### Constructing Triangles

Link to steps → R

Side, Angle, Angle



Side, Angle, Side



Side, Side, Side



# YEAR 9 — REASONING WITH NUMBER... Numbers

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## What do I need to be able to do?

- By the end of this unit you should be able to:
- Identify integers, real and rational numbers
  - Work with directed number
  - Solve problems with number
  - Find HCF/ LCM
  - Add/ Subtract fractions
  - Multiply/ Divide fractions
  - Write numbers in standard form

## Keywords

- Integer:** a whole number that is positive or negative  
**Rational:** a number that can be made by dividing two integers  
**Irrational:** a number that cannot be made by dividing two integers  
**Inverse operation:** the operation that reverses the action  
**Quotient:** the result of a division  
**Product:** the result of a multiplication  
**Multiples:** found by multiplying any number by positive integers  
**Factor:** integers that multiply together to get another number

## Integers, real and rational numbers

Rational – root word: ratio

Real numbers:  $\frac{2}{3}$  stems from 2 |  $\frac{2}{3}$  of the whole

Irrational numbers:  $\sqrt{2}$  the solution is a decimal that never ends and does not repeat

The square root of a negative is not a real number and cannot be found

## HCF/LCM

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

HCF = 6

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

## Standard form

Any number between 1 and less than 10  $\rightarrow A \times 10^n$   $\leftarrow$  Any integer

$$6 \times 10^5 + 8 \times 10^5$$

$$= 600000 + 800000$$

$$= 1400000$$

$$= 1.4 \times 10^6$$

$$(1.5 \times 10^5) \div (0.3 \times 10^3)$$

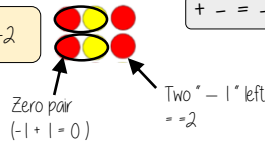
$$15 \div 0.3 \times 10^5 \div 10^3$$

$$= 5 \times 10^2$$

## Directed number

Addition

$$2 + -4 = -2$$

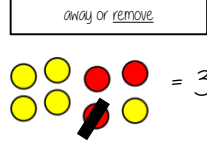


Generalisation  
+ - = -

Subtraction

$$2 - -1 = 3$$

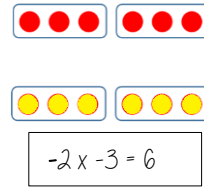
Representation for calculation



Generalisation  
- - = +

Start with the representation of 2

Multiplication



Divisions are the inverse operations



$$a = 5$$

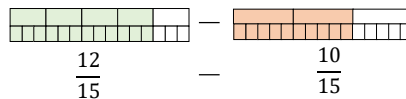
$$b = -4$$

Brackets around negative substitutions helps remove calculation errors

$$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$$

## Addition/ Subtraction of fractions

$$\frac{4}{5} - \frac{2}{3}$$



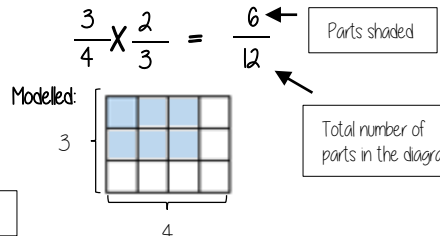
$$= \frac{2}{15}$$

Use equivalent fractions to find a common multiple for both denominators

## Multiplication/ Division of fractions

$$\frac{3}{4} \times \frac{2}{3}$$

This many columns      This many rows



$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

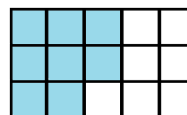
Remember to use reciprocals

$$2 \div \frac{3}{4}$$

$$2 \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

Represented



$$= \frac{8}{3}$$

# YEAR 9 — REASONING WITH NUMBER... Using Percentages

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

## Keywords

- Percent:** parts per 100 — written using the % symbol  
**Decimal:** a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.  
**Fraction:** a fraction represents how many parts of a whole value you have.  
**Equivalent:** of equal value.  
**Reduce:** to make smaller in value.  
**Growth:** to increase/ to grow.  
**Integer:** whole number, can be positive, negative or zero.  
**Invest:** use money with the goal of it increasing in value over time (usually in a bank).  
**Multiplier:** the number you are multiplying by.  
**Profit:** the income take away any expenses/ costs.

## FDP Equivalence

Percentage  
100% = a whole = 100 hundredths

One Whole = 1

10 hundredths  
10 out of 100  
10%

One hundredth  
(one whole split into 100 equal parts)

$\frac{10}{100} = \frac{1}{10} = 0.10$

ones	tenths	hundredths
	.	

## Converting FDP

70/100

This also means 70 - 100

70 out of 100 squares  
70 "hundredths"  
= 7 "tenths"  
0.7

70 hundredths = 70%

Using a calculator

Convert to a decimal

× 100 converts to a percentage

Be careful of recurring decimals  
eg  $\frac{1}{3} = 0.3333333$   
 $\frac{1}{3} = 0.\dot{3}$   
The dot above the 3

## Percentage Increase/ Decrease

Decrease

100%

42% Decrease by 58%

Increase

100%

Increase by 12%

Multiplier Less than 1  
 $100 - 0.58 = 0.42$

Multiplier More than 1  
 $100\% + 12\% = 112\%$   
 $100 + 0.12 = 1.12$

## Percentage change

I bought a phone for £200  
A year later sold it for £125

100%

£200

£125

All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200} \times 100 = 37.5\%$

## Reverse Percentages

40% of my number is 16  
What am I thinking of?

Original Number (100%)

16

40% = 16  
10% = 4  
100% = 40

140% of my number is 84. What is the original number?

Original Number (100%)

84

140% = 84  
10% = 6  
100% = 60

Try to scale down to 10% or 1% and then scale back up to 100%

$$\frac{\text{Difference in values}}{\text{Original value}} \times 100$$

I bought a house for £180,000, I later sold it for £216,000

100%

£180,000

Percentage profit

Money made (profit value)  $\rightarrow \frac{36000}{180000} \times 100 = 20\%$

# YEAR 9 — REASONING WITH NUMBER... Maths & Money

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

## Keywords

- Credit:** money being placed into a bank account  
**Debit:** money that leaves a bank account  
**Balance:** the amount of money in a bank account  
**Expense:** a cost/ outgoing  
**Deposit:** an initial payment (often a way of securing an item you will later pay for)  
**Multiplier:** a number you are multiplying by. (Multiplier more than 1 = increasing, less than 1 = decreasing)  
**Per Annum:** each year  
**Currency:** the type of money a country uses.  
**Unitary:** one — the cost of one.

## Bills and Bank Statements

**Bills** — tell you the amount items cost and can show how much money you need to pay.

Some can include a total  
 Look for different units  
 (Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

## Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19 <sup>th</sup> Sept	Salary	£1500		£1500
19 <sup>th</sup> Sept	Mortgage		£600	£900
25 <sup>th</sup> Sept	Bday Money	£15		£915

## Simple Interest

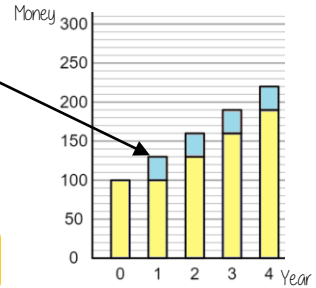
For each year of investment the interest remains the same

$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account  
 e.g Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = \text{£}120$$

This account earned **£120** interest.  
 At the end of year 4 they have **£220**



## Compound Interest

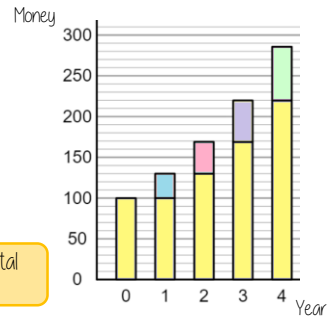
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = \text{£}285.61$$

This account has **£285.61** in total at the end of the 4 years.



## Value Added Tax (VAT)

VAT is payable to the government by a business in the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

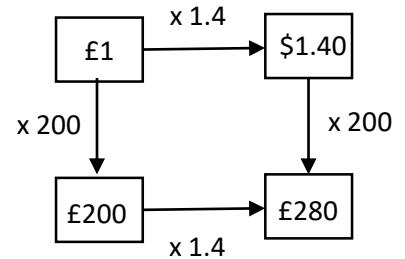
## Wages and Taxes

Salaries fall into tax brackets — which means they pay this much each month from their salary.

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:  
 Time and a half — means 1.5 times their hourly rate  
 Double — 2 times their hourly rate

## Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

### Common Currencies

United Kingdom	£	Pounds
United States of America	\$	Dollars
Europe	€	Euros

## Unit Pricing

4 Oranges £1	5 cupcakes £1.20
-----------------	---------------------

$$\begin{array}{l} 4 = \text{£}1.00 \\ 2 = \text{£}0.50 \\ 1 = \text{£}0.25 \end{array} \left. \begin{array}{l} \div 2 \\ \div 2 \end{array} \right\} \begin{array}{l} 5 = \text{£}1.20 \\ 1 = \text{£}0.20 \end{array}$$

Cost per Unit

To calculate unit per cost you divide by the cost.

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.



# YEAR 9 — REASONING WITH GEOMETRY... Deduction

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

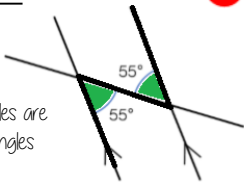
- Identify angles in parallel lines
- Solve angle problems
- Make conjectures with angles
- Make conjectures with shapes

## Keywords

- Parallel:** two straight lines that never meet with the same gradient  
**Perpendicular:** two straight lines that meet at  $90^\circ$   
**Transversal:** a line that crosses at least two other lines.  
**Sum:** the result of adding two or more numbers.  
**Conjecture:** a statement that might be true but is not proven  
**Equation:** a statement that says two things are equal  
**Polygon:** a 2D shape made from straight edges.  
**Counterexample:** an example that disproves a statement

## Alternate angles

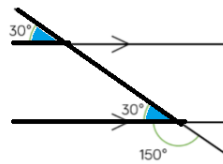
Because alternate angles are equal the highlighted angles are the same size



R

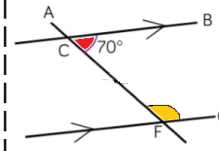
## Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



R

## Co-interior angles



Because co-interior angles have a sum of  $180^\circ$  the highlighted angle is  $110^\circ$

As angles on a line add up to  $180^\circ$  co-interior angles can also be calculated from applying alternate/ corresponding rules first

R

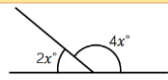
## Solving angle problems

### Angles on a straight line



- Vertically opposite angles**  
Equal
- Angles around a point**  
 $360^\circ$

Link angle facts to algebra



$$2x + 4x = 180^\circ$$

Form an equation

State the reason

The sum of angles on a straight line is  $180^\circ$

Solve

$$2x + 4x = 180^\circ$$

$$6x = 180^\circ$$

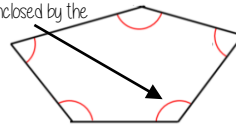
$$x = 30^\circ$$



**Triangles**  
Sum of angles is  $180^\circ$   
Isosceles have the same base angles

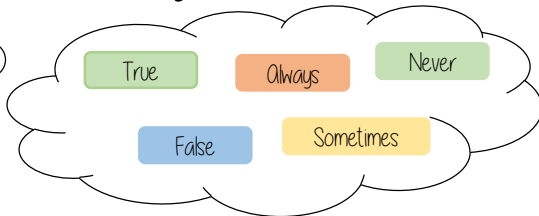
### Interior Angles

The angles enclosed by the polygon



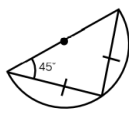
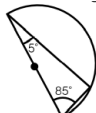
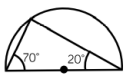
$$(\text{number of sides} - 2) \times 180$$

## Making conjectures with angles



### Proving a conjecture

A pattern is noticed for many cases



Apply the angle rules

The sum of angles in a triangle is  $180^\circ$

Test the theory

$$180 - 70 - 20 = 90$$

$$180 - 85 - 5 = 90$$

$$180 - 45 - 45 = 90$$

Make conjecture

The angle that meets the circumference in a semi circle is  $90^\circ$

### Disproving a conjecture

Only one counterexample is needed to disprove a conjecture

## Making conjectures with shapes

Keywords and facts to recall with shape

- Area: the amount of space inside a shape
- Perimeter: the length around a shape
- Regular Polygons: All sides and angles are equal

### Quadrilateral Facts

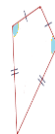
**Square**  
All sides equal size  
All angles  $90^\circ$   
Opposite sides are parallel

**Rectangle**  
All angles  $90^\circ$   
Opposite sides are parallel

**Rhombus**  
All sides equal size  
Opposite angles are equal



**Parallelogram**  
Opposite sides are parallel  
Opposite angles are equal  
Co-interior angles



**Kite**  
No parallel lines  
Equal lengths on top sides  
Equal lengths on bottom sides  
One pair of equal angles

# YEAR 9 — REASONING WITH GEOMETRY... Rotation & Translation

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

## Keywords

**Rotate:** a rotation is a circular movement

**Symmetry:** when two or more parts are identical after a transformation

**Regular:** a regular shape has angles and sides of equal lengths

**Invariant:** a point that does not move after a transformation

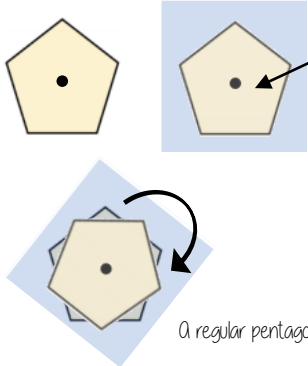
**Vertex:** a point two edges meet

**Horizontal:** from side to side

**Vertical:** from up to down

## Rotational Symmetry

Tracing paper helps check rotational symmetry



1 Trace your shape (mark the centre point)

2 Rotate your tracing paper on top of the original through  $360^\circ$

3 Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

## Translation and vector notation

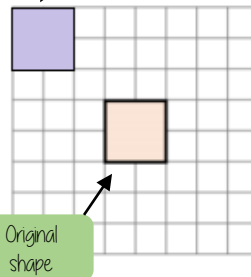
Vector Notation

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

How far left or right to move  
Negative value (left)  
Positive value (right)

How far up or down to move  
Negative value (down)  
Positive value (up)

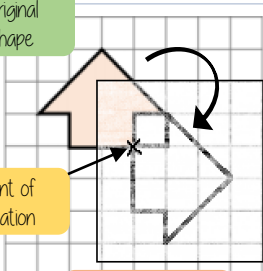
Translation  $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$



Every vertex has been translated by the same amount

## Rotate from a point (in a shape)

Original shape



Point of rotation

Image  $90^\circ$  clockwise

1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

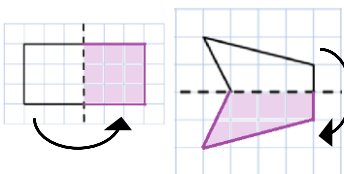
3 Draw the new shape



Clockwise

Anti-Clockwise

## Compare rotations and reflections



**R**

Reflections are a mirror image of the original shape

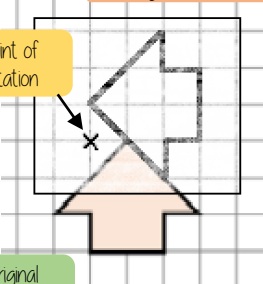
Information needed to perform a reflection:

- Line of reflection (Mirror line)

## Rotate from a point (outside a shape)

Image  $90^\circ$  anti-clockwise

Point of rotation



Original shape

1 Trace the original shape (mark the point of rotation)

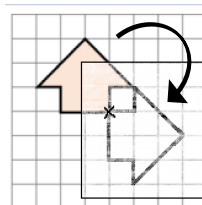
2 Keep the point in the same place and turn the tracing paper

3 Draw the new shape

Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:

- Point of rotation
- Direction of rotation
- Degrees of rotation



# YEAR 9 — REASONING WITH GEOMETRY... Pythagoras' theorem

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

## Keywords

**Square number:** the output of a number multiplied by itself

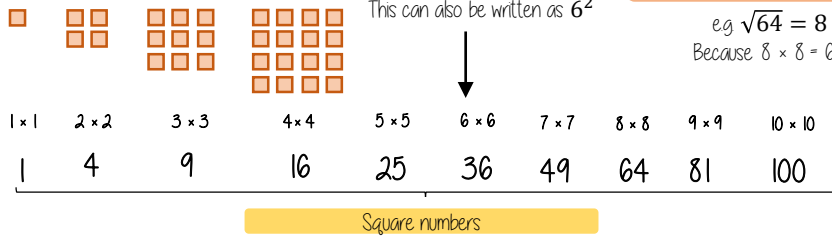
**Square root:** a value that can be multiplied by itself to give a square number

**Hypotenuse:** the largest side on a right angled triangle. Always opposite the right angle.

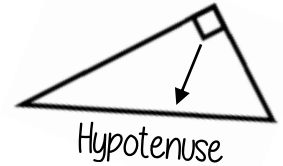
**Opposite:** the side opposite the angle of interest

**Adjacent:** the side next to the angle of interest

## Squares and square roots

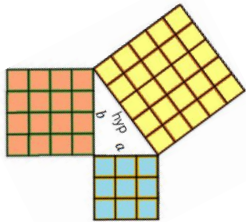


## Identify the hypotenuse



The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.

## Determine if a triangle is right-angled



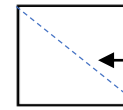
If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

eg  $a^2 + b^2 = \text{hypotenuse}^2$

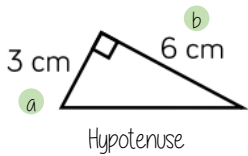
$$\begin{aligned} 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \end{aligned}$$

Substituting the numbers into the theorem shows that this is a right-angled triangle



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

## Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1 Substitute in the values for a and b

$$3^2 + 6^2 = \text{hypotenuse}^2$$

$$9 + 36 = \text{hypotenuse}^2$$

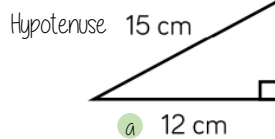
$$45 = \text{hypotenuse}^2$$

2 To find the hypotenuse square root the sum of the squares of the shorter sides

$$\sqrt{45} = \text{hypotenuse}$$

$$6.71\text{cm} = \text{hypotenuse}$$

## Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$

1 Substitute in the values you are given

$$144 + b^2 = 225$$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

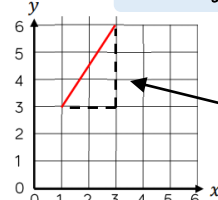
Square root to find the length of the side

$$b^2 = 111$$

$$b = \sqrt{111} = 10.54\text{ cm}$$

## Pythagoras' theorem on a coordinate axis

Find the length of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram

The line segment is the hypotenuse

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle

Be careful to check the scale on the axes

# YEAR 9 — REASONING WITH GEOMETRY... Enlargement & Similarity

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes.

## Keywords

**Similar Shapes:** shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.

**Scale Factor:** the multiple describing how much a shape has been enlarged

**Enlarge:** to change the size of a shape (enlargement is not always making a shape bigger)

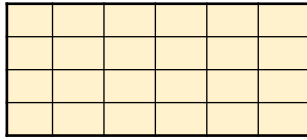
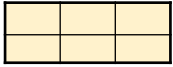
**Corresponding:** objects (or sides) that appear in the same place in two similar situations.

**Image:** the picture or visual representation of the shape

## Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

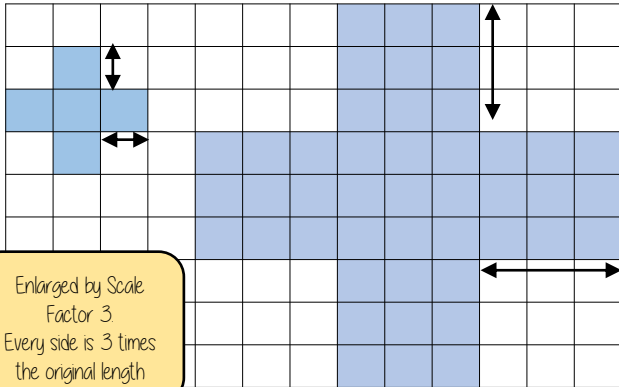
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

## Enlarge by a positive scale factor

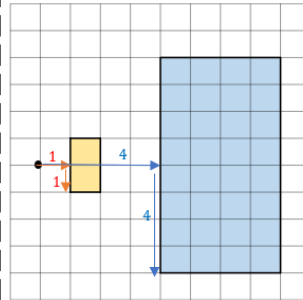
With a scale factor larger than 1 it makes the shape bigger



Enlarged by Scale Factor 3  
Every side is 3 times the original length

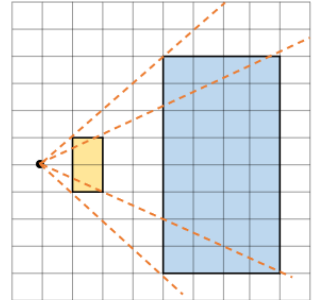
## Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

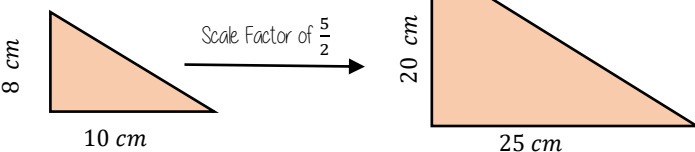
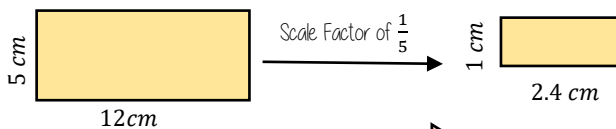
Rays method



Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

## Positive fractional scale factor

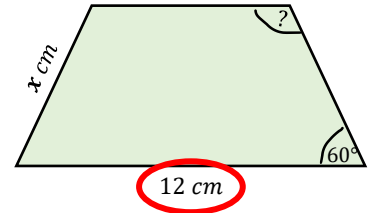
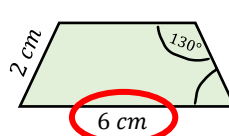
With a scale factor between 0 and 1 it makes the shape smaller



## Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



Corresponding sides identify the scale factor

$$\frac{12}{6} = 2$$

Scale Factor = 2

Calculate the missing side

Length (corresponding side)  $\times$  scale factor  
 $2\text{ cm} \times 2$   
 $x = 4\text{ cm}$

Enlargement does not change angle size

Calculate the missing angle

Corresponding angles remain the same  
 $130^\circ$

# YEAR 9 — REASONING WITH GEOMETRY...

## Solving ratio & proportion problems

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

### Keywords

**Proportion:** a comparison between two numbers

**Ratio:** a ratio shows the relative size of two variables

**Direct proportion:** as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

**Inverse proportion:** as one variable is multiplied by a scale factor the other is divided by the same scale factor.

### Direct Proportion

As one variable changes the other changes at the same rate.

R



4 cans of pop = £2.40

4 cans of pop = £2.40  
2 cans of pop = £1.20

This multiplier is the same in the same way that this would be for ratio

This is a multiplicative change

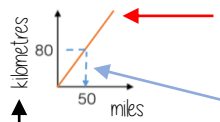
4 cans of pop = £2.40  
12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first e.g. 1 can of pop = £0.60

### Conversion Graphs

Compare two variables

R



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare — then find the associated point by using your graph  
Using a ruler helps for accuracy  
Showing your conversion lines help as a "check" for solutions

Labelling of both axes is vital

### Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

T	1	2	8
G	40	20	5

Annotations: 1 to 2 is  $\times 2$ , 2 to 8 is  $\times 4$ , 40 to 20 is  $\div 2$ , 20 to 5 is  $\div 4$

### Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts



Shop A

4 cans for £1.20

£1.20 ÷ 4

Cost per item

1 can is £0.30  
Or 30p

Shop B

3 cans for 93p

£0.93 ÷ 3

1 can is £0.31  
Or 31p

Shop A is the best value as it is 1p cheaper per can of pop



Shop A

4 cans for £1.20

4 ÷ £1.20

Cost per pound

£1 buys 3.333 cans of pop

3 cans for 93p

3 ÷ £0.93

£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

### Sharing a whole into a given ratio

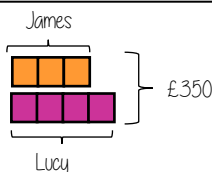
R

James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

Model the Question

James: Lucy

3 : 4



£350 ÷ 7 = £50

□ = one part = £50

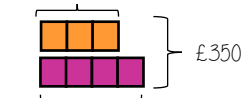
Find the value of one part

Whole: £350  
7 parts to share between  
(3 James, 4 Lucy)

Put back into the question

James: Lucy

James = 3 × £50 = £150



Lucy = 4 × £50 = £200

(x 50) 3 : 4 (x 50)  
£150 : £200

### Finding a value given 1:n (or n:1)

R

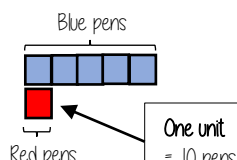
Inside a box are blue and red pens in the ratio 5:1. If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

□ = one part = 10 pens

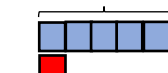


Put back into the question

Blue: Red

(x 10) 5 : 1 (x 10)  
50 : 10

Blue pens = 5 × 10 = 50 pens



Red pens = 1 × 10 = 10 pens

There are 50 Blue Pens

# YEAR 9 — REASONING WITH GEOMETRY... Rates

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Solve speed, distance, time questions
- Use distance time graphs
- Solve density, mass, volume problems
- Solve flow problems
- Use flow graphs
- Interpret rates of change and their units

## Keywords

**Convert:** change

**Mass:** a measure of how much matter is in an object. Commonly measured by weight

**Origin:** the coordinate (0, 0)

**Volume:** the amount of 3D space a shape takes up

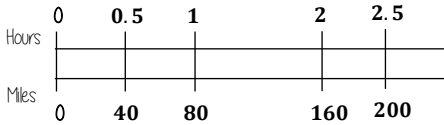
**Substitute:** putting numbers where letters are — replacing numbers into a formula

## Speed, Distance, Time

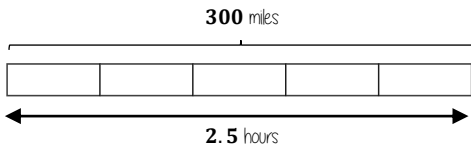
'per' for every  
e.g. 80 miles per hour (mph)  
Travel 80 miles every hour

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours  
It travels 300 miles.



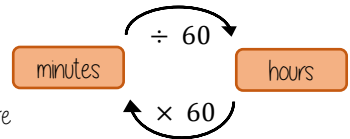
Bar models can help to calculate mph

Each part is half an hour  
Each part is 60 miles

## Speed, Distance, Time



Before calculations — make sure you are working in the same units as the speed



Learn or learn how to rearrange the formula for speed, distance and time

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

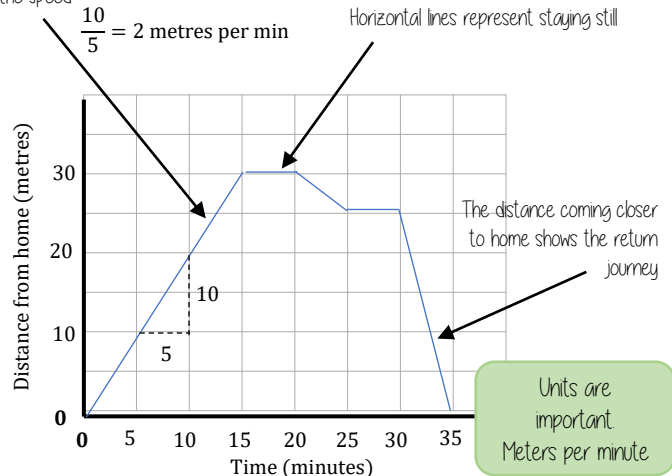
Substitute in the variables given

$$\text{distance} = \text{speed} \times \text{time}$$

## Distance — Time graphs

The steeper a gradient the faster the speed

Gradient = speed

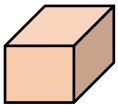


## Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$



$$\text{volume of prism} = \text{Area of cross section} \times \text{Depth}$$



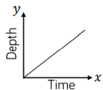
## Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



Units are important  
Ensure any volume calculations are the same unit as the rate of flow

## Rates of change & units

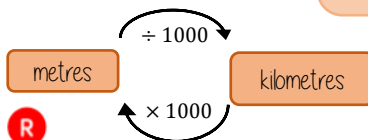
Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles per hour

Exchange rates: euros per pounds

Density: mass per volume



# YEAR 9 — REPRESENTATIONS...

# Probability

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

## Keywords

**Probability:** the chance that something will happen

**Relative Frequency:** how often something happens divided by the outcomes

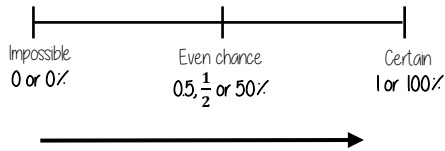
**Independent:** an event that is not effected by any other events.

**Chance:** the likelihood of a particular outcome.

**Event:** the outcome of a probability — a set of possible outcomes.

**Biased:** a built in error that makes all values wrong by a certain amount.

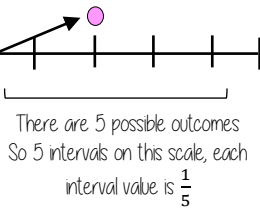
## The probability scale



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability



R

## Single event probability

Probability is always a value between 0 and 1



The probability of getting a blue ball is  $\frac{1}{5}$   
 $\therefore$  The probability of NOT getting a blue ball is  $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$$



R

## Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

## Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

An experiment is carried out 400 times

Show that dark chocolate is expected to be selected 60 times

$$0.15 \times 400 = 60$$

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections

$$\text{Relative frequency} \times \text{Number of times} \\ 0.3 \times 100 = 30$$

## Independent events



The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

$$\text{Probability of event 1} \times \text{Probability of event 2}$$



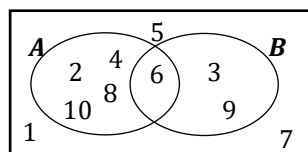
$$P(5) = \frac{1}{6} \quad P(R) = \frac{1}{4}$$

Find the probability of getting a 5 and a red

$$P(5 \text{ and } R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$$

## Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables



	Car	Bus	Wak	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from tossing a coin

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

# YEAR 9 — REPRESENTATIONS...

## Algebraic Representation

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Draw quadratic graphs
- Interpret quadratic graphs
- Interpret other graphs including reciprocals
- Represent inequalities

### Keywords

**Quadratic:** a curved graph with the highest power being 2. Square power.

**Inequality:** makes a non equal comparison between two numbers

**Reciprocal:** a reciprocal is 1 divided by the number

**Cubic:** a curved graph with the highest power being 3. Cubic power.

**Origin:** the coordinate (0, 0)

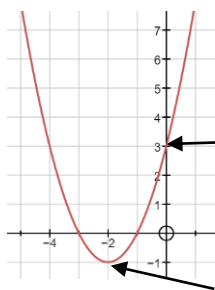
**Parabola:** a 'u' shaped curve that has mirror symmetry

### Quadratic Graphs

$$y = x^2 + 4x + 3$$

If  $x^2$  is the highest power in your equation then you have a quadratic graph

It will have a parabola shape



Substitute the  $x$  values into the equation of your line to find the  $y$  coordinates

$x$	-4	-3	-2	-1	0	1
$y$	3	0	-1	0	3	8

Coordinate pairs for plotting (-3, 0)

Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

### Interpret other graphs

#### Cubic Graphs

$$y = x^3 + 2x^2 - 2x + 1$$

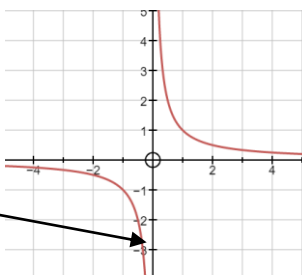
If  $x^3$  is the highest power in your equation then you have a cubic graph



Reciprocal graphs never touch the  $y$  axis  
This is because  $x$  cannot be 0  
This is an asymptote

#### Reciprocal Graphs

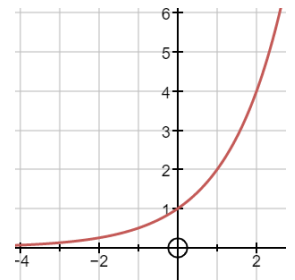
$$y = \frac{1}{x}$$



#### Exponential Graphs

$$y = 2^x$$

Exponential graphs have a power of  $x$

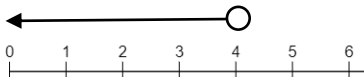


### Represent Inequalities

Multiple methods of representing inequalities

$$x < 4$$

All values are less than 4



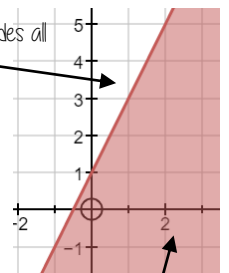
The shaded area indicates all possible values of  $x$



The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$$y \geq 2x + 1$$



The shaded area indicates all possible solutions to this inequality