MOMENTS

KEY WORDS & DEFINITIONS

1. Moment
The turning effect of a force on a rigid body.

2. Resultant Moment
The sum of all moments acting on a rigid body.

3. The Point of Tilting
The instantaneous situation where the reaction at any support or the tension in any supporting string or wire, other than at the pivot, will be zero.

4. Coplanar Forces
Forces that act in the same plane.

5. Lamina
A 2D object whose thickness can be ignored.

MODELLING ASSUMPTIONS & IMPLICATIONS

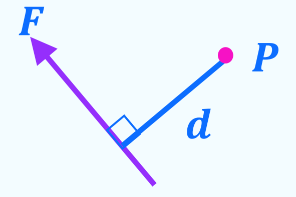
A plank is uniform $\Rightarrow$ Weight acts at the centre of the plank

A plank is a rod $\Rightarrow$ The plank remains straight

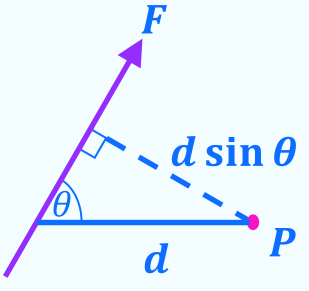
Any people/objects are particles $\Rightarrow$ Their weight acts at the end of any rod

FORMULAE

Moment about P = magnitude of force x perpendicular distance of the force from P



Moment of **F** about P = | **F** | x d **N**m clockwise



Moment of **F** about P = **F**dSinθ **N**m clockwise

WHAT DO I NEED TO KNOW

1. The **units** of Moments are **Newton metres Nm**

2. The **direction** of the Moment (clockwise or anticlockwise) must be included with a moment's value.

3. When a rigid body is in **equilibrium**, the **resultant force** in any direction is 0**N** and the **resultant moment** about any point is 0**N**m

4. The centre of mass of a **non-uniform rod** is **not** necessarily at the **midpoint** of the rod.

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FORCES & FRICTION

KEY WORDS & DEFINITIONS

1. **Friction**
A force which opposes motion.

2. **Coefficient of Friction μ**
A measure of how resistant to motion two surfaces are

3. **Limiting Equilibrium**
The point at which there is equilibrium, but friction is at its maximum

FORMULAE

To calculate Maximum Friction:

$$F_{max} = \mu R$$

Where:

F is the frictional force

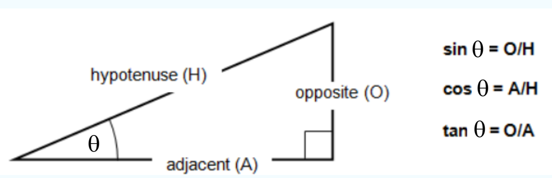
μ is the coefficient of friction

R is the normal reaction between the surfaces.

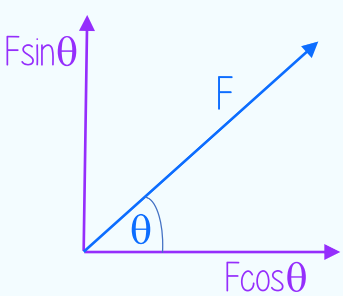
RESOLVING FORCES

"Cos through, Sine away"

Using SOH CAH TOA



when resolving forces gives the following result:



Component $F_x = F\cos\theta$ (through the angle)

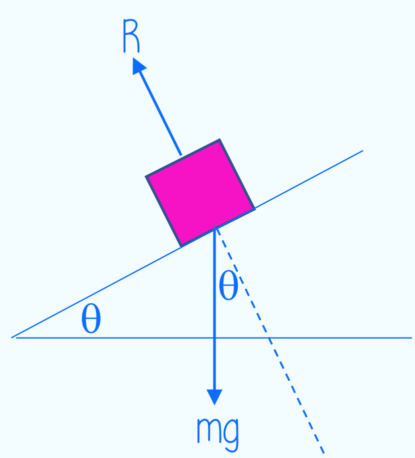
Component $F_y = F\sin\theta$ (away from the angle)

(or $= F\cos(90 - \theta)$)

WHAT DO I NEED TO KNOW

1. If a force is applied at an angle to the direction of motion, resolve it in two perpendicular directions to find the component of force that acts in the direction of motion OR use the triangle law for vector addition.

2. To solve problems on inclined planes, resolve parallel and perpendicular to the plane. REMEMBER, the normal reaction force acts at right angles to the plane, not vertically.



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PROJECTILES

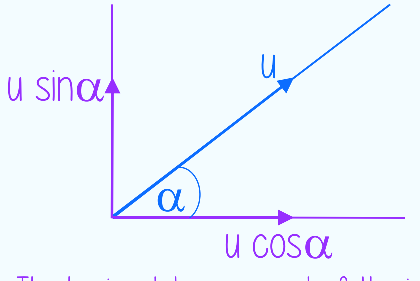
KEY WORDS & DEFINITIONS

1. **Projectile**
   A particle moving in a vertical plane under the action of gravity.
2. **Angle of Projection**
   The initial angle the projectile makes with the horizontal direction.
3. **Speed**
   The magnitude of the velocity, or the resultant velocities.
4. **Range**
   The horizontal distance that the particle travels.
5. **Time of Flight**
   The time taken for the projectile to hit the ground, or other horizontal surface, after being projected.

HORIZONTAL & VERTICAL COMPONENTS OF INITIAL VELOCITY

If a particle is projected with an initial velocity u, at an angle α above the horizontal, α is called 'The angle of projection'.

The velocity can be resolved into components that act horizontally and vertically.



The horizontal component of the initial velocity = u cos α

The vertical component of the initial velocity = u sin α

WHAT DO I NEED TO KNOW

1. The horizontal acceleration of a particle = 0
2. The horizontal velocity of a particle is constant. Therefore s = vt
3. The vertical acceleration a of a partical = g (constant)
4. To find the horizontal & vertical components of the initial velocity, resolve horizontally & vertically
5. When a projectile reaches its maximum height, the vertical component of velocity = 0
6. Acceleration due to gravity = $9.8 m/s^2$
   This does not depend on the mass of the object.
7. The degree of accuracy in your answers must be consistent with the values given in the question. I.e. if g = $10 m/s^2$ in the question, your answer should also be given to 1 sig. fig. Do not leave exact surd answers.
8. Many projectile problems can be solved by first using the vertical motion to find the total time taken.

POSSIBLE EQUATIONS TO DERIVE

For a particle projected with initial velocity U at angle α above horizontal and moving freely under gravity:

- Time of flight $= \dfrac{2U \sin \alpha}{g}$
- Time to reach greatest height $= \dfrac{U \sin \alpha}{g}$
- Range on horizontal plane $= \dfrac{U^2 \sin 2\alpha}{g}$
- Equation of trajectory:

$$y = x \tan \alpha - \frac{gx^2}{2U^2}(1 + \tan^2 \alpha)$$

where y is the vertical height of particle and x is the horizontal distance from the point of projection.

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APPLICATIONS OF FORCES

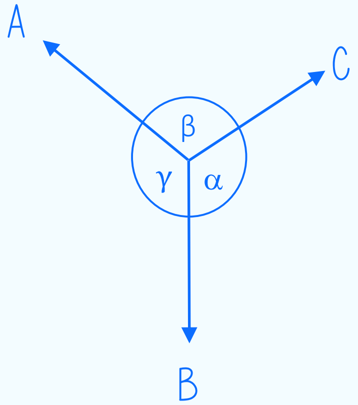
KEY WORDS & DEFINITIONS

Static Equilibrium

A particle is in static equilibrium if it is at rest and the resultant force acting upon it = 0

A rigid body is in static equilibrium if the body is stationary, the resultant force in any direction = 0 and the resultant moment = 0

LAMI'S THEOREM



$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

MODELLING

If a particle is attached separately to two strings, the tension can be different in each string.

If a smooth bead is threaded on a string, the tension in the string will be the same on both sides.

Unless connected particles are moving in the same direction, they must be considered separately.

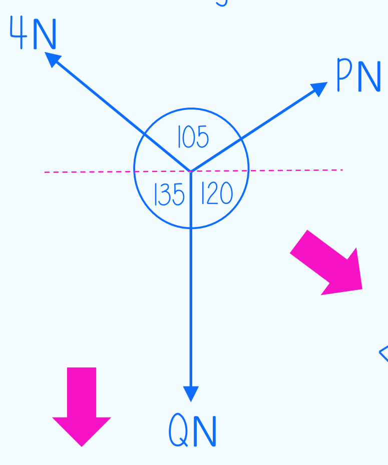
WHAT DO I NEED TO KNOW

1. The maximum value of the frictional force $F_{max} = \mu R$ is reached when the body being considered is on the point of moving. The body is then said to be in 'limiting equilibrium'.

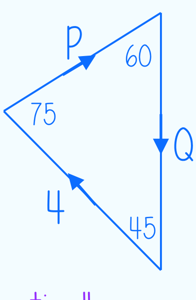
2. In general, the force of friction F is such that $F \leq \mu R$ and the direction of the frictional force is opposite to the direction in which the body would move, if the frictional force were absent.

3. To solve equilibrium problems, draw both a force diagram and a vector diagram.

4. If the angle between forces on a force diagram is θ, the angle between those forces in a triangle of forces is $180^\circ - \theta$. The length of each side of the triangle is the magnitude of the force. (If the particle is not in equilibrium, the vector diagram will not be a closed triangle).



Method 2:
Use Sine Rule



Method 1:
Resolve horizontally & vertically

→ $P\cos 30 - 4\cos 45 = 0$

↑ $P\sin 30 + 4\sin 45 - Q = 0$

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FURTHER KINEMATICS

WHAT DO I NEED TO KNOW

1. To solve problems involving constant acceleration in 2 dimensions, use the SUVAT equations with vector components where
u is the initial velocity
a is the acceleration
v is the velocity at time t (t is a scalar)
r is the displacement at time t

2. To solve problems involving variable acceleration in 2 dimensions, use calculus with vectors by considering each function of time (the vector component) separately.

3. When integrating a vector for a variable acceleration problem, the constant of integration, c, will also be a vector.

4. To find constants of integration, look for initial conditions or boundary conditions.

5. Displacement, velocity & acceleration can be given using *i*-*j* notation, or as column vectors.

FORMULAE

The formula to find **the position vector, r,** of a particle starting at position $\mathbf{r}_0$ that is moving with constant velocity **v** is

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$$

Constant acceleration vector equations:

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$

Calculus for variable acceleration:

Velocity, if displacement is a function of time:

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}$$

$$\int (\boldsymbol{v})\,\mathrm{d}t = \boldsymbol{s}$$

Acceleration, if velocity is a function of time

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{s}}{\mathrm{d}t^2}$$

$$\int (\boldsymbol{a})\,\mathrm{d}t = \boldsymbol{v}$$

DOT NOTATION & DIFFERENTIATING VECTORS

Dot notation is a shorthand for differentiation with respect to time.

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} \qquad \dot{y} = \frac{\mathrm{d}y}{\mathrm{d}t} \qquad \ddot{x} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} \qquad \ddot{y} = \frac{d^2y}{dt^2}$$

To differentiate a vector quantitiy in the form f(t)i + g(t)j, differentiate each function of time separately.

If $r = x\mathrm{i} + yj$, then $v = \frac{\mathrm{d}r}{\mathrm{d}t} = \dot{r} = \dot{x}\mathrm{i} + \dot{y}j$ and $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2r}{\mathrm{d}t^2} = \ddot{r} = \ddot{x}\mathrm{i} + \ddot{y}j$